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16. $\tan^4 x$

Solution:-

By splitting the given function, we have,

$\tan^4 x = \tan^2 x \times \tan^2 x$

Then,

From trigonometric identity, $\tan^2 x = \sec^2 x - 1$

$= (\sec^2 x - 1) \tan^2 x$

By multiplying, we get,

$= \sec^2 x \tan^2 x - \tan^2 x$

Again by using trigonometric identity, $\tan^2 x = \sec^2 x - 1$

$= \sec^2 x \tan^2 x - (\sec^2 x - 1)$

$= \sec^2 x \tan^2 x - \sec^2 x + 1$

Now, integrating on both sides we get,

$$\int \tan^4 x dx = \int \sec^2 x \tan^2 x dx - \int \sec^2 x dx - \int 1.dx$$

$$= \int \sec^2 x \tan^2 x dx - \tan x + x + C$$

Then, let us assume $\tan x = t$

And also assume $\sec^2 x\, dx = dt$

$$\int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

$$\int \tan^4 x dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

17. $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Solution:-

By splitting up the given function,

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

By simplifying, we get,

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

We know that, (sinx/cosx) = tanx and (1/cosx) = secx.

Again, we have (cosx/sinx) = cotx and (1/sinx) = cosecx

= tanx secx + cotx cosecx

Integrating on both the sides, we get

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\tan x \sec x + \cot x \operatorname{cosec} x) dx$$

= secx – cosecx + C